WS #4-5

Properties of Logarithms : $y = \log_a x$ means $x = a^y$

1. Properties of Logarithms: $a>0, a\neq 1, b>0, b\neq 1, M>0, N>0$

① $\log_a 1 = 0$; $\log_a a = 1$ ④ $\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$ ⑦ If $M=N$, then $\log_a M = \log_a N$

② $a^{\log_a M} = M$; $\log_a a^r = r$ ⑤ $\log_a M^r = r\log_a M$ ⑧ If $\log_a M = \log_a N$, then $M=N$

③ $\log_a(MN) = \log_a M + \log_a N$ ⑥ Change of Base Formula $\log_a M = \frac{\log_b M}{\log_b a}$

2. Write as a sum and/or difference of logarithms:

A. $\log_a\left(x\sqrt{x^2+1}\right)$, $x>0$

$\log_a x + \log_a \sqrt{x^2+1} \rightarrow \log_a x + \log_a (x^2+1)^{1/2} \rightarrow \boxed{\log_a x + \frac{1}{2}\log_a(x^2+1)}$

B. $\ln\frac{x^2}{(x-1)^3}$, $x>1$ $\rightarrow \ln x^2 - \ln(x-1)^3 \rightarrow \boxed{2\ln x - 3\ln(x-1)}$

C. $\log_a \frac{\sqrt{x^2+1}}{x^3(x+1)^4}$, $x>0$ $\rightarrow \log_a\sqrt{x^2+1} - \log_a[x^3(x+1)^4]$

$\rightarrow \log_a(x^2+1)^{1/2} - [\log_a x^3 + \log_a(x+1)^4] \rightarrow \boxed{\frac{1}{2}\log_a(x^2+1) - 3\log_a x + 4\log_a(x+1)}$

3. Write as a single logarithm:

A. $\log_a 7 + 4\log_a 3 = \log_a 7 + \log_a 3^4 = \log_a(7\cdot 3^4) = \boxed{\log_a 567}$

B. $\frac{2}{3}\ln 8 - \ln(3^4-8) = \ln 8^{\frac{2}{3}} - \ln(81-8) = \ln\sqrt[3]{8^2} - \ln(73) = \ln 4 - \ln 73$

$= \boxed{\ln\left(\frac{4}{73}\right)}$

C. $\log_a x + \log_a 9 + \log_a(x^2+1) - \log_a 5 = \log_a[x\cdot 9(x^2+1)] - \log_a 5$

$= \boxed{\log_a\left[\frac{9x(x^2+1)}{5}\right]}$

4. Approximate $\log_2 7$ to four decimal places.

$\log_2 7 = y \rightarrow 2^y = 7 \rightarrow \ln 2^y = \ln 7 \rightarrow y\ln 2 = \ln 7 \rightarrow \frac{y\ln 2}{\ln 2} = \frac{\ln 7}{\ln 2}$

$\boxed{y \approx 2.8074}$

5. Change of Base Theorem

A. Show the change of Base theorem for $\log_a M$ $\hookrightarrow \log_a M = \frac{\log_b M}{\log_b a}$

B. $\log_5 89 = \frac{\log 89}{\log 5} \approx \boxed{2.7889}$

C. $\log_{\sqrt{2}}\sqrt{5}$

$\hookrightarrow \frac{\log\sqrt{5}}{\log\sqrt{2}} \approx \boxed{2.3219}$ or $\frac{\ln\sqrt{5}}{\ln\sqrt{2}} = \boxed{2.3219}$

6. Graph $y = \log_2 x$

$\hookrightarrow y = \frac{\log x}{\log 2}$ or $y = \frac{\ln x}{\ln 2}$ $\rightarrow$ graph either on your calculator